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Absorption of TeV Photons and κ-deformed Poincare algebra[1]

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ABSTRACT

We consider the process of collision between a hard photon and a soft photon producing an electron-positron pair, under the assumption that the kinematics be described according to the κ-deformation of the D = 4 Poincare algebra. We emphasize the relevance of this analysis for the understanding of the puzzling observations of multi-TeV photons from Markarian 501. We find a significant effect of the κ-deformation for processes above threshold, while, in agreement with a previous study, we find that there is no leading-order deformation of the threshold condition.

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Recently, there has been increasing interest in tests of phenomenological models describing effects with magnitude set by the Planck length (see, e.g., Refs. [1, 2, 3, 4]). In particular, following the proposal put forward in Ref. [1], Kluzniak [5] (also see the later Ref. [6]) suggested that the universe might be more transparent to multi-TeV photons than expected by conventional relativistic astrophysics. In fact, the phenomenological model of quantum-gravity "space-time foam" considered in Ref. [1] has significant implications [7] for the mechanism of hard-photon disappearence in the far infrared background (FIRB) through electron-positron pair production. It was pointed out already in Ref. [8] that if the spectrum of photons observed from distant astrophysical sources goes significantly beyond 10 TeV we would have to "revise our concepts about the propagation of TeV gamma-rays in intergalactic space". Recent FIRB data from DIRBE [9, 10, 11] and from ISOCOM [12] have been used to render more stringent this 10-TeV limit. As observed by Protheroe and Meyer [13], this result raises a puzzle in light of the fact that HEGRA has detected [14] photons with a spectrum ranging up to 24 TeV from Markarian 501 (a BL Lac object at a distance of $\sim$ 150Mpc).

As shown in Ref. [7] the space-time foam model of Ref. [1] provides a fully consistent explanation of this paradox. We are here interested in an appealing alternative solution of the paradox, based on the role of the Planck length in quantum deformations of the Poincare algebra, rather than in the structure of space-time foam. In particular, we focus on one of the κ-Poincare [15, 16, 17] Hopf algebras[2], in which the deformation parameter, κ, can be naturally (although not necessarily) associated with the Planck length: $\kappa \sim 1/L_p$. A preliminary analysis of the paradox in terms of κ-Poincare was already reported in Ref. [7]. The simplest interpretation of the paradox is in terms of a "threshold anomaly" (a deformed threshold condition for the relevant process) and it might have been desireable to find a significant threshold anomaly following from κ-Poincare; however, it was shown in Ref. [7, 21] that the κ-Poincare threshold anomaly is negligibly small. Still, one can explore other aspects of κ-Poincare in relation with the Markarian-501 paradox. In fact, the evaluation of the optical depth is not only sensitive to the threshold condition: it is also sensitive [7, 21] to the nature of processes somewhat above threshold. It is in this respect that the analysis here reported may

[2] We recall that [18, 19, 20] noncocommutative Hopf algebras characterize quantum deformations of Lie groups and Lie algebras.

turn out to have significant implications for the Markarian-501 paradox: in fact, we show that the kinematics of electron-positron pair production above threshold is significantly affected by the κ deformation.

Because of the significance of the experimental context we make our point within the simplest (least technical) supporting analysis: head-on collision with equipartition of outgoing energy, focusing mostly on the leading order in $1/\kappa$, which is the natural expansion parameter for κ-Poincaré analyses. We also focus on one particular form of κ-deformed Poincaré algebra, written in the so-called "bicrossproduct basis" [16, 17]. Moreover, in our analysis we adopt the conventional quantum-group framework[3] for the role of the κ-Poincaré coproduct [15, 16, 17] in the kinematics of collision processes. The only other κ-Poincaré structure that is relevant for the determination of the κ-deformed kinematic rules for collision processes is the κ-deformed mass-shell condition [15, 16, 17] (dispersion relation).

Let us therefore start by noting here the known formulas for the mass shell and the coproduct. In bicrossproduct basis [16, 17] the κ-Poincaré mass-shell condition is (we set $c = 1$ throughout)

$$\left(2\kappa \sinh\frac{p_0}{2\kappa}\right)^2 - \vec{p}^{\,2} e^{\frac{p_0}{\kappa}} = \left(2\kappa \sinh\frac{m}{2\kappa}\right)^2 \tag{1}$$

and a consistent rule for composition of energy-momentum (coproduct rule) is [16, 17]

$$\vec{p}^{\,(1+2)} = \vec{p}^{\,(1)} + e^{-\frac{p_0^{(1)}}{\kappa}}\, \vec{p}^{\,(2)} \tag{2}$$

$$p_0^{(1+2)} = p_0^{(1)} + p_0^{(2)} \, . \tag{3}$$

Incidentally, we remind the reader that the non–Abelian (noncocommutative) addition law of three-momenta corresponds to the deformation of the dual space–time picture. The noncommutative dual Minkowski space-time coordinates $\hat{x}_\mu = (\hat{x}_i, \hat{x}_0)$ (see [16, 17, 24]) satisfy the relations

$$[\hat{x}_0, \hat{x}_i] = \frac{i}{\kappa}\hat{x}_i \qquad [\hat{x}_i, \hat{x}_j] = 0 \, . \tag{4}$$

[3] Alternative proposals for the role of the coproduct in the kinematics of collision processes have been considered in the context of the theory being developed in Refs. [21, 22, 23].

We are now ready for the analysis of the κ-deformed kinematics of $\gamma + \gamma \rightarrow e^+ + e^-$. The case of interest for the Markarian-501 paradox involves the collision of a hard (energy E, momentum $\vec{P}$) photon and a soft (energy ϵ, momentum $\vec{p}$) photon. We denote with $\vec{p}_+; E_+$ and $\vec{p}_-; E_-$ the energy-momentum of the outgoing electron-positron pair. As announced we focus on head-on collisions ($\vec{P} \cdot \vec{p} = -Pp$) and we adopt the conventional framework [15, 16, 17] for the role of the κ-Poincaré coproduct in the kinematics of collision processes, which amounts to the conservation of the coproduct sum of energy-momentum:

$$\vec{p}^{\,(1+2)}_{in} = \vec{p}^{\,(1+2)}_{out} \tag{5}$$

$$p^{(1+2)}_{0;in} = p^{(1+2)}_{0;out} \; . \tag{6}$$

It is also convenient to introduce the angle θ between the two outgoing momenta: $\vec{p}_+ \cdot \vec{p}_- = p_+ p_- \cos\theta$. In fact, one can use (5) to obtain a relation between the square-moduli, $\vec{p}^{\,(1+2)}_{in} \cdot \vec{p}^{\,(1+2)}_{in} = \vec{p}^{\,(1+2)}_{out} \cdot \vec{p}^{\,(1+2)}_{out}$, which takes the form

$$P^2 - 2Pp \simeq p_+^2 + p_-^2 + 2p_+ p_- \cos\theta - \frac{2}{\kappa} E_+ p_-^2 - \frac{2}{\kappa} E_+ p_+ p_- \cos\theta \; ; \tag{7}$$

where we used the rule (2) with the identifications $\vec{p}^{\,(1)}_{in} = \vec{P}$, $\vec{p}^{\,(2)}_{in} = \vec{p}$, $\vec{p}^{\,(1)}_{out} = \vec{p}_+$, $\vec{p}^{\,(2)}_{out} = \vec{p}_-$; moreover, as announced, we are including only the leading-order κ-dependent corrections, neglecting terms which, in addition to the $1/\kappa$ suppression, are also suppressed by the smallness of θ. Also a term of order p^2 has been dropped on the left-hand side (in the $\gamma + \gamma \rightarrow e^+ + e^-$ processes relevant for the Markarian-501 paradox the energy/momentum scales are such that $p^2 \ll p^3/\kappa$).

Analogously, it is convenient to square (6) obtaining

$$E^2 + 2E\epsilon \simeq E_+^2 + E_-^2 + 2E_+ E_- \; . \tag{8}$$

Up to this point we have only used (coproduct-modified) energy-momentum conservation. We must now enforce the κ-Poincaré mass-shell condition. In the case being studied from (1) it follows that

$$E \simeq P + \frac{1}{2\kappa} P^2 \; ; \quad \epsilon \simeq p \; ; \quad E_\pm \simeq p_\pm + \frac{m_e^2}{2p_\pm} + \frac{1}{2\kappa} p_\pm^2 \; ; \tag{9}$$

where, again, we only included the leading-order κ-dependent corrections and in addition we also took into account (even in the κ-independent terms) the smallness of m_e with respect to p_+ and p_- (certainly well justified for TeV electrons).

Eq. (9) allows to rewrite (7) and (8) as

$$P^2 - 2P\epsilon \simeq p_+^2 + p_-^2 + 2p_+p_-\cos\theta - \frac{2}{\kappa}p_+p_-^2 - \frac{2}{\kappa}p_+^2p_-\cos\theta ; \tag{10}$$

$$P^2 + 2P\epsilon + \frac{1}{\kappa}P^3 \simeq \left(p_+ + p_- + \frac{m_e^2}{2p_+} + \frac{m_e^2}{2p_-} + \frac{1}{2\kappa}p_+^2 + \frac{1}{2\kappa}p_-^2\right)^2 . \tag{11}$$

These equations (10) and (11) establish, for given value of κ, two relations between $P; p_+; p_-; \epsilon$. We observe that Eqs. (10) and (11) show that, while the leading order κ-deformation of the threshold condition vanishes [7], the kinematic conditions for generic processes above threshold are affected by the κ-deformation in leading order. In order to show this feature more explicitly, as announced, we focus on the particular case in which the outgoing energy is equipartitioned between the electron and the positron (also because, in particular, equipartition applies to threshold electron-positron pair production). We can therefore, for the remainder of this paper, adopt the notations $E' \equiv E_+ = E_-$ and $p' \equiv p_+ = p_-$. This allows us to rewrite (10) and (11), for the case of equipartition of outgoing energy, as

$$P^2 - 2P\epsilon \simeq 2p'^2(1+\cos\theta) - \frac{2}{\kappa}p'^3(1+\cos\theta) ; \tag{12}$$

$$P^2 + 2P\epsilon + \frac{1}{\kappa}P^3 \simeq 4p'^2 + 4m_e^2 + \frac{4}{\kappa}p'^3 . \tag{13}$$

The physical content of this result becomes more visible if we combine Eqs. (12) and (13) to obtain the following equivalent (again, to leading order) kinematical conditions

$$p'^2 \simeq \frac{P^2}{3+\cos\theta} - \frac{2m_e^2}{3+\cos\theta} + \frac{1 - 2(1-\cos\theta)(3+\cos\theta)^{3/2}}{6+2\cos\theta}\frac{P^3}{\kappa} \tag{14}$$

$$P\epsilon \simeq \left(1 - \frac{1-\cos\theta}{3+\cos\theta}\right) m_e^2 + \frac{2(1-\cos\theta)}{3+\cos\theta}\frac{P^2}{4} + $$
$$+ \frac{1-\cos\theta}{3+\cos\theta}\left(1 + \frac{4(3+\cos\theta)^2 - 4(1-\cos^2\theta)}{2(3+\cos\theta)^{5/2}}\right)\frac{P^3}{4\kappa} . \tag{15}$$

Eq. (15) is our key result. It is a self-consistent equation that establishes the value of the momentum P of the hard photon required for a head-on collision with a soft-photon of given energy   to produce an electron-positron pair with equipartition of energy and with a given value of the angle   between p_+ and p . The following observations are in order:

(i) In the classical-space-time limit (! 1) our formula (15) of course reproduces the corresponding classical special-relativistic kinematical conditions for pair production with equipartition of outgoing energy.

(ii) For pair production at threshold (= 0) our result predicts an exact cancellation among the leading-order -dependent terms, confirming the earlier result reported in Refs. [7, 21].

(iii) For pair production above threshold our result predicts a non-vanishing leading-order -dependent correction and this correction is such that, for given soft-photon energy , the process would require hard-photon momentum/energy that is higher[4] than the corresponding prediction of classical special-relativistic kinematics. Even for   large enough to satisfy $1/L_p$, the contribution to P coming from the term of order $P^3/(\ )$ can be quite significant ($P/$ is very small, but $P/$ is very large [7]).

(iv) We also emphasize that, while at the coproduct level the identifications $\vec{p}^{\,(1)}_{out} = p_+$, $\vec{p}^{\,(2)}_{out} = p$ are not equivalent to the identifications $\vec{p}^{\,(1)}_{out} = p$, $\vec{p}^{\,(2)}_{out} = p_+$, the final result of our analysis does not depend on this choice.

We leave for future studies (readers from the astrophysics community might be best equipped for this delicate phenomenological analysis) the task of establishing whether the correction we found for pair-production above threshold is sufficient to explain the Markarian-501 paradox. The ingredients for obtaining such an explanation are clearly present in our result: in fact, as mentioned, the evaluation of the optical depth is sensitive [7, 21] to the nature of processes somewhat above threshold (actually, even the peak of the pair-production cross section is somewhat above threshold [7]), and our result suggests that the phase space available for pair-production by a hard photon

[4] It is known [17] that there are two versions of the -deformed Poincare algebra in the bicrossproduct basis, differing by the sign in front of the   parameter. The results here obtained also apply to the other version of the bicrossproduct basis, upon replacing consistently   !   , in which case, of course, the effect we found goes in the opposite direction: for given soft-photon energy the process would require hard-photon momentum/energy that is smaller than the corresponding prediction of classical special-relativistic kinematics.

with given energy E might be reduced in κ-Poincaré, thereby allowing for a modification of the optical depth result.

If indeed future studies will confirm that our result provides a solution of the Markarian-501 paradox it should also be possible to distinguish between this model and other models being considered for a solution of the paradox. The mentioned space-time-foam model of Ref. [1] would interpret the paradox as a "threshold anomaly" while in the model here adopted the deformation of the threshold condition is negligible (but the deformation of processes above threshold is significant). These two alternative pictures should lead to different observable consequences, possibly verifiable in future more refined experiments.

Even easier is the discrimination between the model here adopted and the Coleman-Glashow Lorentz-invariance-violation model [25] which is also known [7,26] to provide a solution to the Markarian-501 paradox. In fact, the Coleman-Glashow model predicts deviations from classical special-relativistic kinematics even in the low-energy regime, whereas in the model here adopted the deformation is completely negligible in low-energy phenomena (if indeed $\kappa \sim 1/L_p$). Sensitive tests of special-relativistic kinematics for low-energy processes could therefore discriminate between the two models.

Whether or not the model here adopted does lead to a solution of the Markarian-501 paradox (and whether or not it proves to be a better solution than its alternatives), our result would remain useful as a characterization of κ-Poincaré Hopf algebras. In almost a decade [15] of research on this subject a large number of results have been obtained, establishing the mathematical properties of the formalism, but only very few characteristic predictions have been identified. The hope of finding an explanation to the Markarian-501 paradox has reenergized research in this direction. We established in this note that a characteristic feature of κ-deformed kinematics is the presence of leading-order corrections to the kinematic rules for processes above threshold, while, as already shown in Ref. [7], there is no leading-order corretion to the threshold condition.

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